Polarization-mode dispersion arises from fiber asymmetry caused by production defects and by external perturbations, which both vary at random. From a phenomenological point of view, according to mode-coupling theory the perturbations can be modeled as a birefringence, which can be conveniently described by the local birefringence vector \( \mathbf{\beta}(z, \omega) \). This vector is defined in the three-dimensional space of Stokes vectors, and it is a function of position along the fiber \( z \) and of optical frequency \( \omega \). Vector \( \mathbf{\beta}(z, \omega) \) contains all the information about perturbations that act on the fiber, and consequently it completely describes all the polarization-mode dispersion sources.

Direct measurement of \( \mathbf{\beta}(z, \omega) \) is extremely difficult; nevertheless, several indirect reflectometric methods were recently proposed, based on polarization-sensitive optical time-domain reflectometry (POTDR), and stimulated Brillouin scattering. These techniques have provided an opportunity to measure the beat length \( L_B \) of a fiber, i.e., the amplitude of the local birefringence.

In Ref. 6 we reported experimental results that showed that the modulus of a linear birefringence is a Rayleigh-distributed random variable, and we pointed out how well this result agrees with the theoretical model proposed by Wai and Menyuk. According to the Wai–Menyuk model (hereafter the WMM), \( \mathbf{\beta}(z) = (\beta_1, \beta_2, \beta_3) \) varies both in modulus and in orientation, as described by the following stochastic differential equations (SDEs):

\[
\frac{d\beta_i}{dz} = -p \beta_i(z) + \sigma \eta_i(z), \quad i = 1, 2,
\]

where it is assumed that \( \beta_3(z) = 0 \). Here \( \eta_1 \) and \( \eta_2 \) are white-noise, Gaussian-distributed, statistically independent processes with zero mean value and unitary standard deviation. Parameters \( p \) and \( \sigma \) describe the statistical properties of \( \beta_i(z) \). It may be shown that, with a suitable choice of the initial condition, \( \beta_i(z)(i = 1, 2) \) are Gaussian-distributed, stationary processes with zero mean and the same standard deviation \( \sigma_\beta = \sigma^2/(2\rho) \). Consequently, the modulus of \( \mathbf{\beta}(z) \) is a Rayleigh-distributed random variable, with mean value \( 2\pi/L_B = \sigma_\beta \sqrt{\pi}/2 \). It is worth noting that the WMM does not provide for circular birefringence \( (\beta_3 = 0) \); however, this is not a severe limitation when one is dealing with standard telecommunication fibers.

By means of the WMM, it is also possible to calculate the correlation of the components of \( \mathbf{\beta}(z) \), which reads (\( i = 1, 2 \)) as (Ref. 7)

\[
r_\beta(z, u) = \mathbb{E}[\beta_i(z)\beta_i(z + u)] = \sigma_\beta^2 \exp(-\rho|u|),
\]

so the correlation length of the birefringence results in \( L_F = 1/\rho \). In this Letter, we give what we believe is the first experimental confirmation of Eq. (2), obtained by means of a POTDR and a novel data elaboration technique. Such a result, besides corroborating the WMM, provides a way to measure \( L_F \). Knowledge of this parameter is useful for a better understanding of the causes of PMD and may help in the design of low-PMD fibers.

POTDR allows the state of polarization (SOP) of the backscattered field to be measured as a function of the scattering point. The evolution of such SOP is governed by the following equation:

\[
\frac{d\delta_B}{dz} = \mathbf{\beta}_B(z) \times \delta_B(z),
\]

where \( \mathbf{\beta}_B(z) \) is the Stokes vector that represents the SOP of the field that is backscattered at point \( z \). The quantity \( \mathbf{\beta}_B(z) \) is the round-trip birefringence vector, and it is related to the fiber local birefringence vector \( \mathbf{\beta}(z) \) according to the following equation:

\[
\mathbf{\beta}_B(z) = 2\mathbf{M}^T(z)\mathbf{L}_F(z) = 2\mathbf{M}^T(z)\mathbf{\beta}(z),
\]

where \( \mathbf{M} = \text{diag}(1, 1, -1) \) is a diagonal matrix, \( \mathbf{R}(z) \) is the Müller matrix of the fiber. \( T \) indicates the transposition, and \( \mathbf{L}_F(z) \) is the linear component of \( \mathbf{\beta}(z) \). In the WMM it is assumed that these two vectors coincide, i.e., that \( \beta_3 = 0 \).
If we measure \( \hat{r}_{BB}(z) \) we can calculate the round-trip birefringence vector and, using Eq. (3), can infer properties of the local birefringence vector. The key point of our analysis is the following expression, which gives the correlation of \( \beta_{B1}(z) \) (\( i = 1, 2, 3 \)):

\[
\rho_{BB}(z, u) = \sigma_{BB}^2 \exp(-\rho|u|),
\]

where \( \sigma_{BB} = \sqrt{8/3} \). Comparing this result with Eq. (2), we see that \( \rho_{BB}(z, u) \) has the same decay rate as \( \rho_{B}(z, u) \). Hence, by measuring the correlation of \( \beta_{B1}(z) \), we can determine the correlation of \( \beta_{1}(z) \) and hence of \( L_F \). Moreover, we can also determine beat length \( L_B \) because, from Ref. 6, \( L_B = 8\sqrt{\pi}/3/\sigma_{BB} \).

Preliminary numerical evidence for Eq. (4) was given in Ref. 9. We now provide a theoretical demonstration also, using the SDE theory. For simplicity, we consider \( \beta_{B1}(z) \), the first component of \( \beta_{B}(z) \); the other two components may be analyzed in the same way. If we indicate by \( \hat{c}(z) = (c_1, c_2, c_3) \) the first column of \( \mathbf{R}(z) \), we can write \( \beta_{B1} = 2\hat{c} \cdot \hat{B} = 2(c_1\beta_1 + c_2\beta_2) \). Moreover, \( \hat{c}(z) \) satisfies the following equation:

\[
d\hat{c}/dz = \hat{B}(z) \times \hat{c}(z),
\]

with the initial condition that \( \hat{c}(0) = (1, 0, 0) \); this equation together with Eq. (1) forms the SDE that describes our problem:

\[
\frac{d}{dz} \begin{pmatrix} \beta_1 \\ \beta_2 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ \eta_1 & \eta_2 \end{pmatrix} \begin{pmatrix} -\rho \beta_1 \\ -\rho \beta_2 \\ -\beta_2 c_3 - \beta_1 c_2 \end{pmatrix}.
\]

(5)

Given the particular SDE that we are considering, there is no difference between the choices of Itô and Stratonovich, so Eq. (5) yields the following infinitesimal generator:

\[
\mathcal{A} = \beta_2 c_3 \frac{\partial}{\partial c_1} - \beta_1 c_3 \frac{\partial}{\partial c_2} + (\beta_1 c_2 - \beta_2 c_1) \frac{\partial}{\partial c_3} - \rho \left( \beta_1 \frac{\partial}{\partial \beta_1} + \beta_2 \frac{\partial}{\partial \beta_2} \right) + \frac{\sigma^2}{2} \left( \frac{\partial^2}{\partial \beta_1^2} + \frac{\partial^2}{\partial \beta_2^2} \right).
\]

(6)

Let us now consider the correlation \( \rho_{BB}(z, u) \) and assume that \( u \gg 0 \). We define \( E_z[X] = E[X|\beta_{B1}(z)] \) as the conditional average of \( X \), so the derivative of \( \rho_{BB}(z, u) \) with respect to \( u \) can be calculated as follows:

\[
\frac{\partial \rho_{BB}(z, u)}{\partial u} = \frac{\partial}{\partial u} E_z[\rho_{BB}(z, \beta_{B1}(z + u))]
\]

\[
= E_z[\rho_{BB}(z, \beta_{B1}(z + u))];
\]

(7)

the first equality is given by the properties of the conditional average, and the second holds because \( \beta_{B1}(z) E_z[\rho_{BB}(z, \beta_{B1}(z + u))] \) is a deterministic process with respect to \( u \). To calculate the derivative of \( E_z[\rho_{BB}(z + u)] \) we use Dynkin’s formula, which yields

\[
\frac{\partial E_z[\rho_{BB}(z + u)]}{\partial u} = E_z[(\mathcal{A} \rho_{BB})(z + u)],
\]

(8)

and, applying the generator, we get

\[
\frac{\partial \rho_{BB}(z, u)}{\partial u} = -\rho \rho_{BB}(z, u),
\]

(9)

with the initial condition that \( \rho_{BB}(z, 0) = E[\rho_{BB}(z)] \). Strictly speaking, \( E[\rho_{BB}(z)] \) is not independent of \( z \), but when \( z \gg L_F \) it reaches a steady state for which \( E[\rho_{BB}(z)] = \sigma_{BB}^2 = (8/3\sigma_{BB}^2) \). Now Eq. (9) can be written as

\[
\frac{\partial}{\partial u} E_z[\rho_{BB}(z, u)] = E_z[(\mathcal{A} \rho_{BB})(z + u)],
\]

(10)

and

\[
\frac{\partial}{\partial u} E_z[\rho_{BB}(z, u)] = -\rho \rho_{BB}(z, u),
\]

(11)

with the initial condition that \( \rho_{BB}(z, 0) = E[\rho_{BB}(z)] \). Strictly speaking, \( E[\rho_{BB}(z)] \) is not independent of \( z \), but when \( z \gg L_F \) it reaches a steady state for which \( E[\rho_{BB}(z)] = \sigma_{BB}^2 = (8/3\sigma_{BB}^2) \). Now Eq. (9) can be written as

\[
\frac{\partial}{\partial u} E_z[\rho_{BB}(z, u)] = E_z[(\mathcal{A} \rho_{BB})(z + u)],
\]

(12)

and

\[
\frac{\partial}{\partial u} E_z[\rho_{BB}(z, u)] = -\rho \rho_{BB}(z, u),
\]

(13)

with the initial condition that \( \rho_{BB}(z, 0) = E[\rho_{BB}(z)] \). Strictly speaking, \( E[\rho_{BB}(z)] \) is not independent of \( z \), but when \( z \gg L_F \) it reaches a steady state for which \( E[\rho_{BB}(z)] = \sigma_{BB}^2 = (8/3\sigma_{BB}^2) \). Now Eq. (9) can be written as

\[
\frac{\partial}{\partial u} E_z[\rho_{BB}(z, u)] = E_z[(\mathcal{A} \rho_{BB})(z + u)],
\]

(14)

and

\[
\frac{\partial}{\partial u} E_z[\rho_{BB}(z, u)] = -\rho \rho_{BB}(z, u),
\]

(15)

with the initial condition that \( \rho_{BB}(z, 0) = E[\rho_{BB}(z)] \). Strictly speaking, \( E[\rho_{BB}(z)] \) is not independent of \( z \), but when \( z \gg L_F \) it reaches a steady state for which \( E[\rho_{BB}(z)] = \sigma_{BB}^2 = (8/3\sigma_{BB}^2) \). Now Eq. (9) can be written as
easily be solved for $u \geq 0$, and, recalling that correlations are even functions, we finally obtain Eq. (4). It should be noted that, if circular birefringence were present, Eq. (4) would not hold.

To verify this result, we measured the round-trip birefringence of long single-mode fibers tightly wound about drums. Measurements were performed by means of a POTDR scheme (Fig. 1). An external-cavity laser is used to produce narrow-band, 5-ns pulses centered at 1534 nm. This procedure guarantees a spatial resolution of $\sim 0.5$ m. Pulses are amplified by an erbium-doped fiber amplifier (EDFA) and then filtered by an acousto-optic modulator that opens a temporal window of 50 ns to let the pulse pass while it stops the EDFA spontaneous emission. The backscattered field is passed through a quarter-wave plate, followed by a linear polarizer, and the transmitted power is acquired and processed by a high-resolution commercial optical time-domain reflectometer (OTDR).

The round-trip birefringence vector was calculated according to Ref. 6, and from $\beta_{p}(z)$ we estimated the correlation of its components. Two typical results are shown in Figs. 2 and 3 for G.652 fibers. In both figures continuous curves represent the experimental correlation of $\beta_{p}(z)$ and dashed curves are the theoretical best fits obtained according to Eq. (4). Figure 2 refers to a 16.2-km-long single-mode fiber; its perturbation length is $L_{F} = 5.4$ m, and $\sigma_{\beta B} = 0.3$ m$^{-1}$. Figure 3 refers to one of the innermost fibers of a four-fiber ribbon. It shows a long perturbation length, $L_{F} = 20.4$ m, with $\sigma_{\beta B} = 0.6$ m$^{-1}$. In both cases, good agreement has been reached between experimental and theoretical results.

A more detailed set of results is summarized in Table 1. The rows refer to five different fibers, each of which was tightly wound about a drum. Values of the perturbation length, measured according to Eq. (4), are listed in the third column. The fourth and the fifth columns show the beat-length values: the former refers to measurements performed according to Ref. 6; the latter, to values obtained with Eq. (4). As can be seen, there is good agreement between the two sets of data.

It is well known that $L_{B}$, $L_{F}$, and the mean differential group delay (DGD) depend on one another; this relationship can be expressed as follows:

$$
\langle \Delta \tau \rangle^{2} = \frac{1}{3} \left( \frac{8 \lambda L_{F}}{\pi c L_{B}} \right)^{2} \left[ \exp\left(-\frac{L}{L_{F}}\right) + \frac{L}{L_{F}} - 1 \right],
$$

where $L$ is the fiber length, $\lambda$ is the wavelength, and $c$ is the speed of light in vacuum. Using Eq. (10), we can predict the mean DGD from the measurements of $L_{B}$ and $L_{F}$ performed at a fixed wavelength. Of course, this calculated DGD value should coincide with the result obtained by direct DGD measurement. We verified this correspondence by measuring $\langle \Delta \tau \rangle$ for each fiber, using Jones matrix eigenanalysis. As can be seen from Table 1, there is a fairly good correspondence between the measured values of the DGD (sixth column) and the calculated values (seventh column).

In summary, a new technique to measure the correlation length of fiber birefringence has been reported. Experimental data show that the round-trip birefringence vector has an exponential correlation, which agrees with theoretical predictions of the WMM.

A. Galtarossa and L. Palmieri thank the Information Society Technologies ATLAS project and the Ministero dell’Università e delle Ricerca Scientifica e Tecnologica 40% project (prot. 9909198913) for supporting their activity. L. Palmieri’s e-mail address is luca.palmieri@wave.dei.unipd.it.

### Table 1. Measurements of $L_{B}$, $L_{F}$, and $\langle \Delta \tau \rangle$

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>Fiber Length (m)</th>
<th>$L_{F}$ (m)</th>
<th>From Eq. (6)</th>
<th>From Eq. (4)</th>
<th>Meas. (ps)</th>
<th>Calc. (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.652</td>
<td>13,200</td>
<td>8.2</td>
<td>21.1</td>
<td>21.2</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>G.652</td>
<td>14,700</td>
<td>15.4</td>
<td>29.6</td>
<td>29.4</td>
<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
<td>G.653</td>
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<td>19.4</td>
<td>18.9</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>G.653</td>
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<td>14.9</td>
<td>15.1</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>G.655</td>
<td>19,200</td>
<td>5.0</td>
<td>23.2</td>
<td>23.0</td>
<td>0.07</td>
<td>0.10</td>
</tr>
</tbody>
</table>

References