Influence of the input state of polarization on interferometric and polarimetric measurements of birefringence

T. Tambosso and S. Donati

Department of Electronics, University of Pavia, 27100 Pavia, Italy

Received June 10, 1988; accepted February 24, 1989

Using a general model to schematize polarimetric and interferometric measurements, we derive the dependence of the output signal on the input state of polarization. We show that in any birefringent medium, in addition to the eigenvectors that maximize the interferometric phase signal, there exist dual vectors that maximize the amplitude of the polarimetric signal. The dual vectors lie on the maximum circle normal to the eigenvector axis.

The evolution of the state of polarization (SOP) is an important issue for coherent communications and optical fiber sensors. In interferometric sensors and coherent receivers, fluctuations in the output SOP induce the so-called polarization fading of the photodetected signal,[1–4] for which polarization controllers[2,4] and polarization diversity have been devised. In polarimetric fiber sensors[5] fluctuations of the SOP directly reflect themselves as a measurement error whose effect has not yet been analyzed in detail. Also, it is well known that interferometric and polarimetric measurements are complementary because they involve the phase and the amplitude, respectively, of the signal. As shown below, for those input SOP's maximizing the information in one measurement, the other measurement carries no information.

The aim of this Letter is to generalize the analysis of interferometric and polarimetric measurements in birefringent media so as to find which SOP is best suited to perform the measurement and which error is eventually generated.

First, let us model the *interferometric measurement* as in Fig. 1(A), where the input SOP is described by the Jones vector $\mathbf{V}_i$. After propagation the output vector $\mathbf{V}_u$ is combined onto the photodetector with the input delayed by an arbitrary phase ϕ_0. The current signal at the photodetector output is then

$$i \propto |\mathbf{V}_i \exp(i\phi_0) + \mathbf{V}_u|^2 = |\mathbf{V}_i|^2 + |\mathbf{V}_u|^2 + 2\,\mathrm{Re}[\mathbf{V}_i \exp(i\phi_0)\mathbf{V}_u{}^*]. \quad (1)$$

The interferometric signal, contained in the last term of relation (1), is the real part of the scalar product P between the input and output SOP's,

$$P = \mathbf{V}_i \exp(i\phi_0) \cdot \mathbf{V}_u{}^*. \quad (2)$$

Now we explicitly define $\mathbf{V}_i$ in terms of the eigenvectors (or polarization modes) of the medium $\mathbf{A}_1$ and $\mathbf{A}_2$ as

$$\mathbf{V}_i = R_1\mathbf{A}_1 + R_2\mathbf{A}_2, \quad (3)$$

where the coefficients R_1 and R_2 are complex numbers (given by the scalar product of $\mathbf{V}_i$ with $\mathbf{A}_1$ and $\mathbf{A}_2$). With $\mathbf{A}_1$ and $\mathbf{A}_2$ normalized ($|\mathbf{A}_1| = |\mathbf{A}_2| = 1$) and $\mathbf{A}_1 \cdot \mathbf{A}_2 = 0$, we have

$$|R_1|^2 + |R_2|^2 = 1. \quad (4)$$

Now let us write the output vector $\mathbf{V}_u$ as

$$\mathbf{V}_u = \mathbf{J} \cdot \mathbf{V}_i, \quad (5)$$

where $\mathbf{J}$ is the Jones matrix of the medium. By inserting Eq. (3) into Eq. (5), since $\mathbf{J} \cdot \mathbf{A}_1 = \lambda_1\mathbf{A}_1$ and $\mathbf{J} \cdot \mathbf{A}_2 = \lambda_2\mathbf{A}_2$ for the eigenvectors and $\lambda_1 = \lambda_2{}^* = \exp(i\Delta z)$ for the eigenvalues λ_1 and λ_2 of a Jones matrix $\mathbf{J}$ of a nondissipative medium (with the determinant equal to unity), we get easily for $\mathbf{V}_u$

$$\mathbf{V}_u = R_1\mathbf{A}_1 \exp(i\Delta z) + R_2\mathbf{A}_2 \exp(-i\Delta z), \quad (6)$$

where z is the path length and Δ is the birefringence coefficient of the medium. [Specifically, $\Delta = \beta_l/2$ for a linear birefringence, $\Delta = \beta_c/2$ for a circular birefringence, $\Delta = (1/2)(\beta_l{}^2 + \beta_c{}^2)^{1/2}$ for a coexisting linear and circular birefringence, etc.] Combining Eqs. (2), (3), and (6), one has

$$P = [\cos \Delta z + i(|R_2|^2 - |R_1|^2)\sin \Delta z]\exp(i\phi_0), \quad (7)$$

and the interferometric signal therefore reads

$$\mathrm{Re}\,P = \cos \Delta z \cos \phi_0 - (|R_2|^2 - |R_1|^2)\sin \Delta z \sin \phi_0. \quad (8)$$

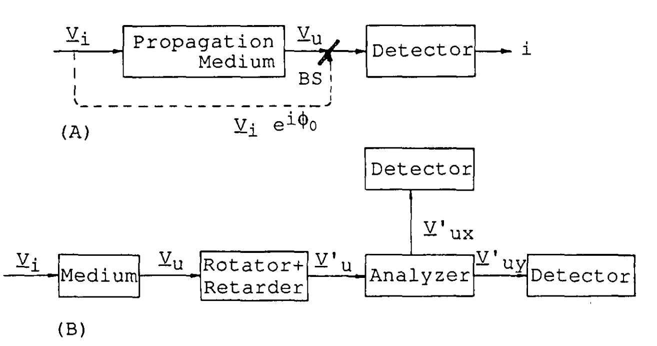

Fig. 1. Models of (A) interferometric and (B) polarimetric measurements. BS, beam splitter.

0146-9592/89/100476-03$2.00/0

To discuss the dependence of the interferometric signal from the input SOP, we first note that if one of the eigenvectors of the medium is chosen as the input ($R_1 = 1$ and $R_2 = 0$ or vice versa) the result is

$$\mathrm{Re}\, P = \cos(\phi_0 \pm \Delta z), \tag{9}$$

as expected for a measurement of a phase shift $\pm\Delta z$ with an added reference phase ϕ_0. For all other input SOP's (R_1 and R_2 different from zero) the signal is smaller than the above value, and in the case of $|R_1| = |R_2|$ it becomes

$$\mathrm{Re}\, P = \cos \Delta z \cos \phi_0, \tag{10}$$

i.e., it is modulated in amplitude by the bias term cos ϕ_0. This is the extreme case of maximum fading of polarization.[1,4] Alternatively, Eqs. (9) and (10) can be interpreted in terms of phase- and amplitude-modulated signals. This is easily seen by letting $\phi_0 = \omega t$, so that the above results are also extended to the case of a heterodyne detection scheme. If an eigenvector $\mathbf{A}_1$ or $\mathbf{A}_2$ is used as the input, the beating (or interferometric) signal given by Eq. (9) contains the birefringence information Δz as a phase-modulated term of the carrier ωt, and there is no information in amplitude. The reverse is true if the SOP's with $|R_1| = |R_2|$ are used as input: the information on Δz is contained in the amplitude-modulation term cos Δz of the carrier cos ωt, and there is no information in phase.

We call these SOP's, obtained by equal weighting ($|R_1| = |R_2|$) of the eigenvectors $\mathbf{A}_1$ and $\mathbf{A}_2$, the *dual vectors* of the eigenvectors. On the Poincaré sphere the dual vectors lie on the maximum circle normal to the axis connecting the eigenvectors.

Since the propagation is described by a rotation[6–8] of the Poincaré sphere around the eigenvector axis, an input dual vector will change its SOP on propagation but still be a dual vector [this can also be seen by comparing Eqs. (3) and (6)]. The intersections of the dual-vector circle and of the Poincaré sphere equatorial circle give the two linearly polarized dual vectors of the medium. If $2\Phi_A$ and $2\Psi_A$ are the coordinates (inclination and ellipticity) of the eigenvectors, those of the two linear dual vectors are $2\Phi = \pm\pi/2 + 2\Phi_A$ and $2\Psi = 0$. For a medium presenting a distributed linear (β_l) and circular (β_c) birefringence, in which the rotation velocity[7,8] of the Poincaré sphere is $\Omega = \Omega_l + \Omega_c$ with $|\Omega| = (\beta_l{}^2 + \beta_c{}^2)^{1/2}$, we show in Fig. 2 the eigenvectors $\mathbf{A}_1$ and $\mathbf{A}_2$ and the locus of the dual vectors (circle D). Here the angular coordinates[6,7] of mode $\mathbf{A}_2$ are $2\Phi_A = 0$ (inclination) and $2\Psi_A = 2 \arctan \alpha$ (ellipticity), where $\alpha = (\beta_c/2)/(\beta_l/2 + \Delta)$ and $\Delta = (\beta_l{}^2 + \beta_c{}^2)^{1/2}/2$.

Further insight can be obtained by expressing the scalar product P [Eq. (2)] in the form

$$P = |P| \exp(i\phi), \tag{11}$$

so that from Eq. (7) one finds that

$$|P|^2 = 1 - 4|R_1|^2|R_2|^2 \sin^2 \Delta z, \tag{12}$$

$$\phi = \arctan[(|R_2|^2 - |R_1|^2)\tan \Delta z] + \phi_0. \tag{13}$$

For the eigenvectors ($R_1 = 0$ and $R_2 = 1$ or vice versa)

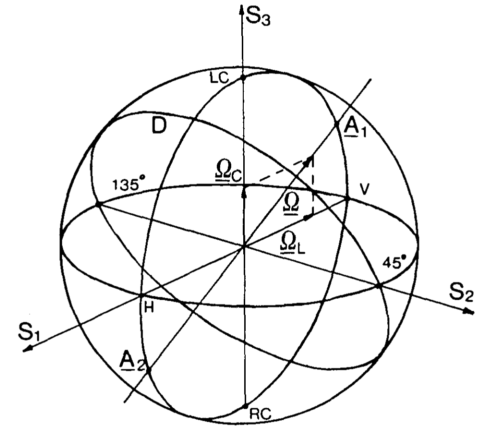

Fig. 2. Representation of eigenvectors $\mathbf{A}_1$ and $\mathbf{A}_2$ and dual vectors (circle D).

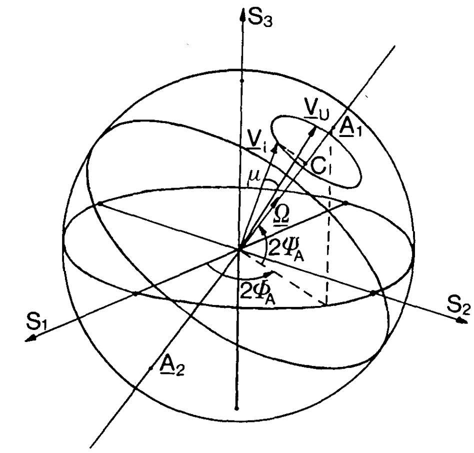

Fig. 3. Geometrical interpretation of the scalar product P and of weights R_1 and R_2.

we have $|P| = 1$ and $\phi = \pm\Delta z + \phi_0$, while for the dual vectors ($|R_1| = |R_2| = 1/\sqrt{2}$) we have $|P| = \cos \Delta z$ and $\phi = \phi_0$. On the Poincaré sphere the modulus $|P|$ of the scalar product has a simple geometrical interpretation related to the angle μ between the input and output SOP vectors (Fig. 3). It can be shown[9] that

$$|P| = |\mathbf{V}_i \cdot \mathbf{V}_u{}^*| = \cos \mu/2. \tag{14}$$

The projection of $\mathbf{V}_i$ on the eigenstates axis intercepts two segments $\overline{A_1C}$ and $\overline{A_2C}$ that are proportional to $|R_1|^2$ and $|R_2|^2$ (Ref. 10) (Fig. 3). The phase ϕ is connected to $|R_1|^2$ and $|R_2|^2$ through Eq. (13) but has no direct geometrical interpretation on the Poincaré sphere, because ϕ/z is in general dependent on z.

Both the modulus and the phase of the scalar product P depend in general on the phase shift variation Δz [Eqs. (12) and (13)]. These two quantities have the meaning of the information content carried by the amplitude and phase, respectively. We may maximize the information contained in the phase by requiring that the modulus is independent of Δz or, equivalently, that

$$\partial|P|/\partial\Delta z = 0. \tag{15}$$

By inserting Eq. (12) into Eq. (15) we obtain as a condition $|R_1|\,|R_2| = 0$, which is satisfied just by the eigenvectors; conversely, if we maximize the information contained in the amplitude by letting

$$\partial\phi/\partial\Delta z = 0, \tag{16}$$

we obtain the condition $|R_1| = |R_2|$, which is the one defining the dual vectors.

Last, we note that the above analysis is readily extended to the case of birefringence in both arms of the interferometer. For this case we can go back[4] to that of Fig. 1(A) by considering $\mathbf{J}\cdot\mathbf{J}_r^{-1}$ as the Jones matrix of the equivalent medium, where $\mathbf{J}_r^{-1}$ is the inverse of the Jones matrix of the reference-arm birefringence.

The general scheme of a *polarimetric measurement* can be modeled as in Fig. 1(B). Here $\mathbf{V}_i$ and $\mathbf{V}_u$ are the Jones vectors at the input and output of the birefringent medium, $\mathbf{V}_u$ may be modified in $\mathbf{V}_u'$ by means of a suitable rotator and retarder combination, and $\mathbf{V}_u'$ is analyzed in its components V_{ux}' and V_{uy}' with respect to axes S_1 and S_2.

Conceptually the polarimetric signal is maximized by applying a rotation and a retardance such that the input vector $\mathbf{V}_i$ is brought to coincide with the S_1 axis and the output vector $\mathbf{V}_u'$ is brought to lie on the equator of the Poincaré sphere ($S_3 = 0$). In view of the equivalence theorem,[6] this is always possible and physically realizable. Since any rotation of the Poincaré sphere leaves the distances unchanged,[6,8] we have

$$\overline{\mathbf{V}_u'S_1} = \overline{\mathbf{V}_u\mathbf{V}_i} = \mu, \tag{17}$$

where μ is obtained by the differential equation of polarization evolution,[6,8] i.e.,

$$\mathrm{d}\mu = |\Omega \times \mathbf{V}|\mathrm{d}z = \Omega \sin \overline{\Omega V}\, \mathrm{d}z, \tag{18}$$

and the angle $\overline{\Omega V}$ can be related to longitudes Φ_Ω and Φ_V and latitudes Ψ_Ω and Ψ_V through spherical trigonometry as

$$\cos \overline{\Omega V} = \cos 2(\Phi_\Omega - \Phi_V)\cos 2(\Psi_\Omega - \Psi_V). \tag{19}$$

Limiting ourselves to the case of constant Ω, so that $\mathbf{V}$ rotates around Ω with a constant angle $\overline{\Omega V}$, and in general letting $|\Omega| = 2\Delta$,[8] we have

$$\mu = 2\Delta z \sin \overline{\Omega V_i}. \tag{20}$$

Since the output $\mathbf{V}_u'$ has coordinates $\Phi = \mu$ and $\Psi = 0$, it can be written as the Jones vector,

$$\mathbf{V}_u' = \begin{bmatrix}\cos \mu/2\\ \sin \mu/2\end{bmatrix} = \begin{bmatrix}\cos \Delta' z\\ \sin \Delta' z\end{bmatrix}, \tag{21}$$

where the birefringence coefficient Δ' is defined explicitly, in terms of input vector angular coordinates Φ_i and Ψ_i, as

$$\Delta' = \Delta[1 - \cos^2 2(\Phi_\Omega - \Phi_i)\cos^2 2(\Psi_\Omega - \Psi_i)]^{1/2}. \tag{22}$$

As special cases of Eqs. (21) and (22), we consider the eigenvectors for which $\mathbf{V}_i \parallel \Omega$, $\Delta' = 0$, and no polarimetric signal is developed, and the dual vectors for which $\mathbf{V}_i \perp \Omega$, $\Delta' = \Delta$, and the output vector has the components $\cos \Delta z$ and $\sin \Delta z$.

Another useful expression can be obtained in terms of the weight coefficients R_1 and R_2 by writing $\mathbf{V}_i$ as in Eq. (3) and $\mathbf{V}_u$ as in Eq. (6). From the projection of the input vector on the eigenvector axis Ω the two segments $\overline{A_1C} = 2|R_2|^2$ and $\overline{A_2C} = 2|R_1|^2$ are found, which allow us to evaluate $\sin \overline{\Omega V}$ as $(1 - \cos^2 \overline{\Omega V})^{1/2} = [1 - (1 - \overline{A_1C})^2]^{1/2} = 2|R_1|\,|R_2|$. Therefore we can rewrite Eq. (22) in the form

$$\Delta' = 2\Delta|R_1|\,|R_2|. \tag{23}$$

It is interesting to note that, with the rotator and retarder compensation, the output signal $\mathbf{V}_u'$ in a polarimetric measurement can always be brought to the form of cos and sin components of $\Delta' z$. The reduction factor $\Delta'/\Delta = 2|R_2|\,|R_1|$ is independent of the argument of R_2 and R_1 or, equivalently, from the longitude of $\mathbf{V}$ with respect to Ω; Δ'/Δ is unity for the dual vectors and decreases as it moves on the Poincaré sphere toward the eigenvectors, for which it is zero.

In conclusion, we have derived expressions regarding interferometric and polarimetric measurements in terms of input SOP's. We have shown that the dual vectors maximize the polarimetric amplitude and give a zero output for the interferometric output, as opposed to the eigenmode SOP's, for which the reverse is true.

References

1. D. W. Stowe, D. R. Moore, and R. G. Priest, IEEE J. Quantum Electron. **QE-18,** 1644 (1982).
2. T. Okoshi, IEEE J. Lightwave Technol. **LT-5,** 44 (1987).
3. K. H. Wanser and N. H. Safar, Opt. Lett. **12,** 217 (1987).
4. A. D. Kersey, A. Dandridge, and A. B. Tveten, Opt. Lett. **13,** 288 (1988).
5. T. G. Giallorenzi, J. Bucaro, A. Dandridge, G. Siegel, J. H. Cole, S. Rashleigh, and R. G. Priest, IEEE J. Quantum Electron. **QE-18,** 626 (1982).
6. P. S. Theocaris and E. E. Gdoutos, *Matrix Theory of Photoelasticity* (Springer-Verlag, New York, 1979).
7. S. C. Rashleigh, IEEE J. Lightwave Technol. **LT-1,** 312 (1983).
8. R. Ulrich and A. Simon, Appl. Opt. 18, 2241 (1979).
9. R. Ulrich, Opt. Lett. **1,** 109 (1977).
10. Y. Yen and R. Ulrich, Appl. Opt. **20,** 2721 (1981).